Dispersion Analysis of TLM Node for Modeling General Anisotropic and Gyromagnetic Materials

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Abstract

In this paper the dispersion relation for the three-dimensional TLM condensed node for modeling materials with arbitrary permittivity and permeability tensors is presented, and the dispersion error associated with the TLM model for these media is studied. A full dispersion analysis of the TLM node is performed when modeling sapphire substrates and gyromagnetic material. The study lays the foundation for minimizing the numerical dispersion error when modeling such media.

1.0 Introduction

Among the critical issues regarding the accuracy of TLM results are the errors due to frequency and directional dispersion in the discrete spatial TLM network. It is well known that the TLM method has a cutoff frequency and deviations from the linear dispersion relation for frequencies approaching the cutoff frequency. This error is produced by the space discretization. In order to evaluate the accuracy of the TLM method and quantify the dispersion error, the dispersion relation of the discrete mesh must be known. The generalized dispersion analysis of two- and three-dimensional TLM schemes was first proposed by Nielsen and Hoefer in 1993 [1]. The usual approach for the derivation of the TLM dispersion relation is based on Floquet's theorem [2] and the network structure. Further work on the dispersion behavior of isotropic media modeled by condensed nodes has been reported by Krumpholz and Russer [3] and Trenkic et al. [4]. The dispersion in anisotropic media has been studied by Huber et al. [5] and Trenkic [6], but the investigations are restricted to wave propagation along the mesh axes and the mesh diagonals in symmetric anisotropic media.

Recently, Huang and Wu have proposed a unified TLM model for wave propagation in electrical and optical structures considering permittivity and permeability tensors [7]. Using this model a TLM algorithm has been derived to address various computational issues in non-diagonal tensor problems encountered in practical situations. However, the wave properties of the node employed in this model have not been systematically investigated yet. The study of the dispersion characteristics of this node remains a great challenge because of the complexity and size of calculations involved. In the paper the dispersion analysis is based on the numerical solution of an eigenvalue equation to overcome this difficulty.

2.0 Theory

The eigenvalue equation of dispersion in the TLM mesh is an implicit function of the propagation constant k_0 of the travelling wave and of the mesh propagation constants k_x, k_y and k_z along the x, y and z directions.

$$det(I - TPS) = 0 \qquad (1)$$

where I is a 18×18 identity matrix, S is the scattering matrix of the TLM node and P is a matrix which contains the mesh propagation constants. Its elements are zero except for

$$P_{1,12} = P_{5,7} = e^{jk_y \Delta y}$$

$$P_{2,9} = P_{4,8} = e^{jk_z \Delta z}$$

$$P_{3,11} = P_{6,10} = e^{jk_x \Delta x}$$

$$P_{7,5} = P_{12,1} = e^{-jk_y \Delta y}$$

$$P_{8,4} = P_{9,2} = e^{-jk_z \Delta z}$$

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$$P_{10,6} = P_{11,3} = e^{-jk_x\Delta x}$$

$$P_{13,13} = P_{14,14} = P_{15,15} = 1$$

$$P_{16,16} = P_{17,17} = P_{18,18} = -1$$

T is a matrix which contains the network propagation constant. Δl is the smallest dimension of a cuboid cell.

$$T = e^{-jk_o\Delta l}I \quad (2)$$

To obtain the dispersion relation, a MATLAB program has been developed to generate a numerical solution for the dispersion characteristics.

3.0 Dispersion characteristics of the TLM node modeling different media

Since sapphire substrates have low dielectric losses, low temperature coefficients of dielectric constant and linear expansion along with high thermal conductivity, they are very attractive for microwave devices. Consider a microstrip line printed on a m-cut sapphire substrate (Fig. 1). The relative permittivity tensor is given in the microstrip coordinate system by

$$\hat{\varepsilon} = \begin{bmatrix} \varepsilon_{xx} & \varepsilon_{xy} & 0 \\ \varepsilon_{yx} & \varepsilon_{yy} & 0 \\ 0 & 0 & \varepsilon_{zz} \end{bmatrix} \quad (3)$$

where

$$\varepsilon_{xx} = \varepsilon_1 \cos^2\theta + \varepsilon_2 \sin^2\theta$$

$$\varepsilon_{yy} = \varepsilon_1 \sin^2\theta + \varepsilon_2 \cos^2\theta$$

$$\varepsilon_{xy} = \varepsilon_{yx} = (\varepsilon_1 - \varepsilon_2)\sin\theta\cos\theta$$

$$\varepsilon_{zz} = \varepsilon_1$$

ε_1 and ε_2 are equal to 9.4 and 11.6 [7], respectively. An angle, $\theta = 60^o$, is arbitrarily selected, and the dispersion characteristics of the TLM node modeling the m-cut sapphire substrate is shown in Fig. 3. Two different spatial discretizations where used. $\Delta x = 0.2$, $\Delta y = \Delta z = 0.1$ (cuboid cell) yield the dashed line, and $\Delta x = \Delta y = \Delta z = 0.1$ (cubic cell) the solid line. The normalized propagation vector is plotted in a polar representation in Fig. 3. The vector k_0 describes the unit circle for the infinitesimal mesh. When a coarse discretization $\Delta x/\lambda = 0.15$ is selected, the wavelength in the TLM network can no longer be considered large compared with the network parameter Δx, and the velocity becomes dispersive and depends on the direction of propagation. The maximum dispersion occurs in the direction at the angle of 35^o. For comparison, the dispersion error of the two different discretizations has been plotted in Fig. 4. It appears that we can choose a wave propagation direction for which the dispersion error is zero.

A similar study has been performed for a microstrip line printed on r-cut sapphire substrate (Fig. 2). The dielectric permittivity tensor [9] for the r-cut ($\theta = 57.6^o$) sapphire is

$$\hat{\varepsilon} = \begin{bmatrix} 9.4 & 0 & 0 \\ 0 & 10.03 & -0.99 \\ 0 & -0.99 & 10.97 \end{bmatrix} \quad (4)$$

Fig. 5 shows its dispersion relation. Unlike in the m-cut sapphire, the higher dispersion occurs along the axial directions, and the minimum error can be found at an angle close to 45^o direction (Fig. 6). Comparing the dispersion characteristics with the two spatial discretizations in Fig. 5, the dispersion error of the cubic cell is less than that of the cuboid cell due to the finer discretization.

The TLM node can model not only materials with permittivity tensor, but also with permeability tensor. Huang and Wu have applied it to simulate a rectangular waveguide partially filled with magnetized ferrite [7] and obtained excellent agreement with the exact solutions [10]. In this case, the permeability tensor of the ferrite material is

$$\hat{\mu} = \mu_o \begin{bmatrix} \mu & 0 & -j\kappa \\ 0 & 1 & 0 \\ j\kappa & 0 & \mu \end{bmatrix} \quad (5)$$

where $\mu = -3.5741$ and $\kappa = -8.2586$.

For this particular case the dispersion characteristic of the partially filled waveguide is shown in Fig. 7. In order to compare with the dispersion relations of the m-cut and r-cut sapphires the same spatial discretization has been used. The magnitude of the normalized propagation vector is plotted vs. the angle with the x-axis. Fig. 8 shows that the dispersion error is very small. The maximum dispersion error is less than 0.8 percent in this case.

4.0 Conclusion

The dispersion relation for the three-dimensional TLM condensed node modeling arbitrary permittivity and permeability tensors is obtained. In the analysis, the dispersive behavior of the TLM node modeling m-cut sapphire, r-cut sapphire substrates and gyromagnetic material is investigated with the same spatial discretization. It is verified that a relatively fine mesh is required to obtain good accuracy in the simulation of the m-cut sapphire material. This analysis shows that it is possible to minimize the numerical dispersion error when modeling anisotropic and gyromagnetic media.

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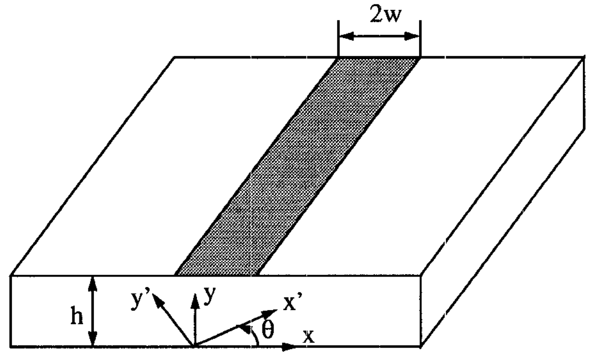

Fig.1 Microstrip geometry showing the crystal (x',y') and microstrip (x,y) axes. (m-cut sapphire)

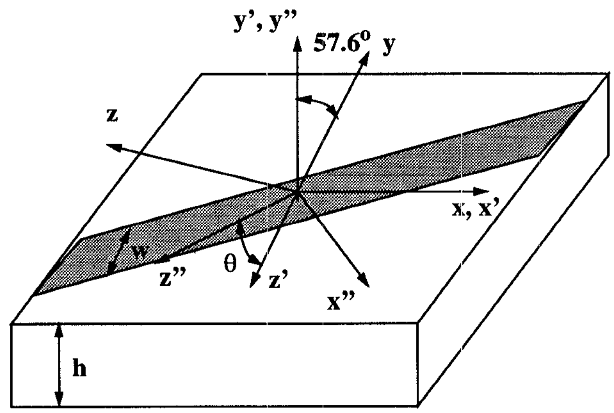

Fig.2 Arbitrarily oriented r-cut sapphire-based microstrip.

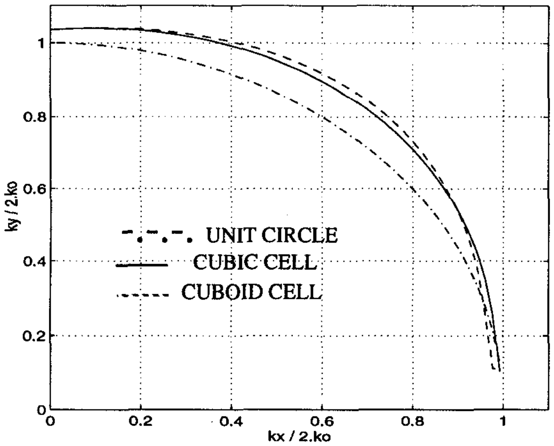

Fig. 3 The dispersion characteristics for a 3D condensed node in x-y plane. Δx = 0.2, Δy = Δz=0.1 (sapphire-m-cut, θ=60°)

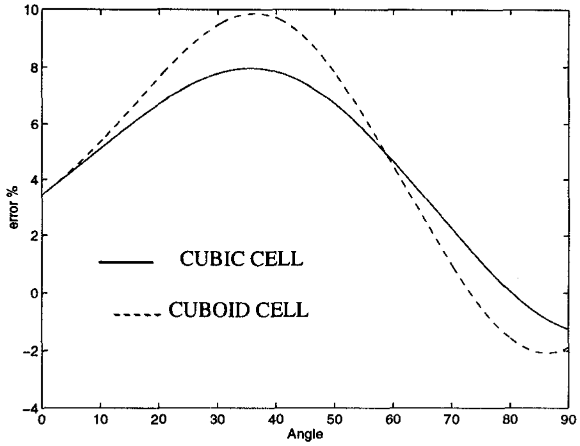

Fig.4 The dispersion error for a 3D condensed node in x-y plane. Δx = 0.2, Δy = Δz = 0.1 (sapphire-m-cut, θ=60°)

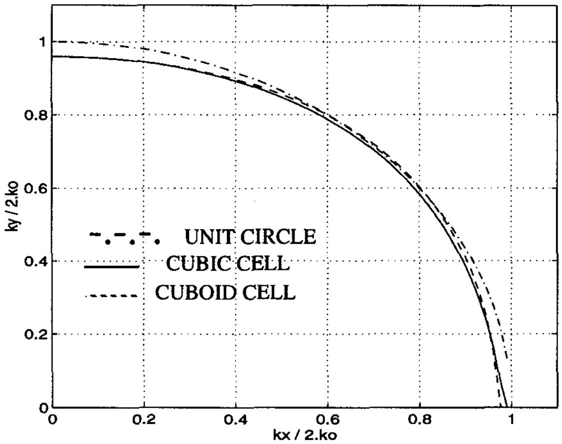

Fig.5 The dispersion characteristics for a 3D condensed node in x-y plane. Δx = 0.2, Δy = Δz = 0.1 (sapphire-r-cut)

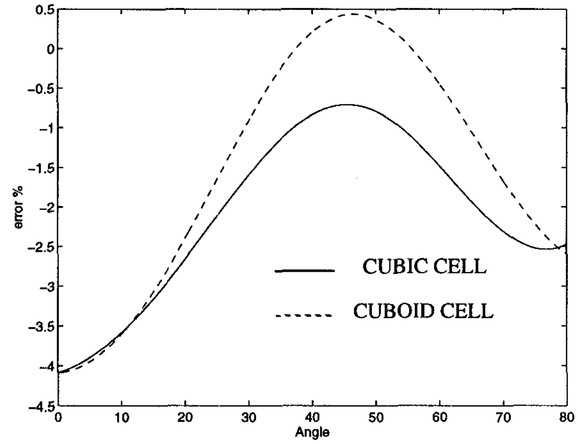

Fig. 6 The dispersion error for a 3D condensed node in x-y plane. Δx = 0.2, Δy = Δz = 0.1 (sapphire-r-cut, θ=57.6°)

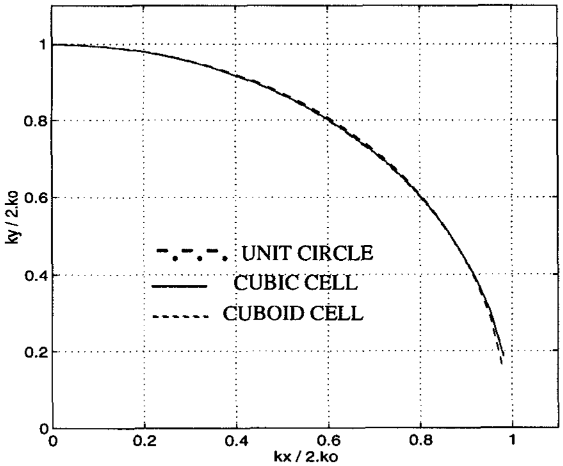

Fig.7 The dispersion characteristics for a 3D condensed node in x-y plane. Δx = 0.2, Δy = Δz = 0.1 (gyromagnetic material)

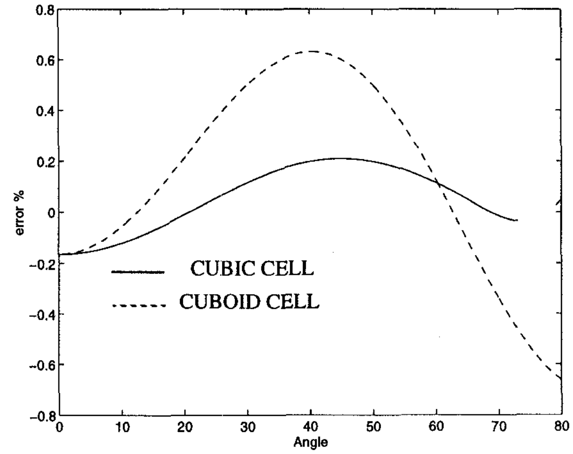

Fig. 8 The dispersion error for a 3D condensed node in x-y plane. Δx = 0.2, Δy = Δz = 0.1 (gyromagnetic material)